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P, T violating magneto-electro-optics

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Abstract. We study linear and bilinear magneto-electro-optical effects due to the propagation of light in centro-symmetric media in the presence of P, T violating interactions and external transverse and longitudinal electric and/or magnetic fields. We show that new magneto-electric optical effects appear. In particular, we show the existence of a Jones birefringence proportional to the square of the transverse field amplitude. All these effects are an unambiguous signature of the P, T violation, and a search for such new phenomena could also provide novel limits on electric dipole moment (EDM) of matter.

PACS. 11.30.Er Charge conjugation, parity, time reversal, and other discrete symmetries – 33.55.Fi Other magnetooptical and electrooptical effects – 42.25.Ja Polarization

1 Introduction

Charge conjugation (C) and parity (P) violation have been observed in nature in weak interactions and in K and B particles decay respectively. The violation of both CP and T induces an electric dipole moment (EDM) in ordinary matter particles such as neutrons and electrons [1]. The first search for such a phenomenon in neutrons is more than 50 years old [2]. Since then, several experiments on fermions, atoms and molecules have been carried on [1]. In particular, present accepted limits on electron and neutron EDM are respectively 1.6×10^{-27} e.cm [3] and 6.3×10^{-26} e.cm [4], and the limit for a permanent electric dipole moment of the ^{199}Hg is 2.1×10^{-28} e.cm [5].

Standard model CP violation predicts very low values for the neutron, proton and electron EDM, but CP violation also appears in new theories such as supersymmetry, giving rise to EDM values that can be tested in experiments [1].

CP violation is also expected in Quantum ChromoDynamics. The fact that this violation has never been observed is called the *strong CP problem*, and a possible solution to this problem could be the existence of a new pseudoscalar boson known as the axion [6].

The role of the time reversal operator T, associated to time symmetry, in the determination of the properties of molecules in electric and magnetic fields is discussed in reference [7], in particular for the case of time-invariant enantiomorphic (*true chiral*) systems, i.e. systems that exist in two distinct enantiomeric states that are interconverted by parity P but not by time reversal T combined with any proper spatial rotation. Enantiomeric states are the distinguishable mirror-images of a physical system. Time-invariant enantiomorphic systems shows natural optical activity like sugar solutions. In general, chirality is always associated to P violation. Since a *true chiral* system do not violate T, by the CPT theorem we see that it must violate C. On other hand, the combination of P and T violations gives time-noninvariant enantiomorphic (*false chiral*) systems i.e. systems that again exist in two distinct enantiomeric states but which are interconverted by time reversal T as well as parity P (see also [8] and references within).

From the phenomenological point of view much attention has been paid to the fact that an electric field applied in the presence of a P, T violation creates a magnetization, and that a magnetic field creates an electrical polarization (see e.g. [9]). Propagation of light in the presence of a P, T violating interaction has been studied by several authors [10–12] to show that a rotation of the polarization plane of light occurs in presence of an electric field parallel to the light propagation vector (see also [13]).

In this letter we study linear and bilinear magneto-electro-optical effects due to the propagation of light in centro-symmetric media in the presence of P, T violating interactions and external transverse and longitudinal electric and/or magnetic fields. We show that new magneto-electric optical effects appear. In particular, we show the existence of a linear birefringence along axes which are at ±45° relative to the transverse magnetic field direction (Jones linear birefringence) proportional to the square of the field amplitude. We also recover the effect already predicted [10–12]. Our study uses mainly a method based on pictorial symmetry arguments introduced in 1980 by de

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Figueiredo and Raab [14]. A model calculation is also presented for a quantum vacuum. All these magneto-electro-optical effects are an unambiguous signature of the P, T violation, and a search for such new phenomena could also provide novel limits on EDM of matter.

2 Pictorial analysis

In 1980 de Figueiredo and Raab [14] have shown that pictorial symmetry arguments can be used to study light propagation. In particular this method has been used to prove the existence of new optical effects like the Jones birefringence [15]. In this approach the effect is studied by comparing schematic pictures representing a possible experiment and its forms when subject to certain space and time transformations. In the following we will be especially concerned by P and T transformations of light polarization states (horizontal ↔, vertical ↕, at 45° ⤢, at −45° ⤡, circular states ↺ and ↻), electric field **E** and magnetic field **B**. These properties can be summarized as follows [14]:

- under P : ↔→↔, ↕→↕, ⤢ → ⤡, ⤡ → ⤢, ↺ → ↻, ↻ → ↺ **E** → −**E**, **B** → **B**;
- under T : ↔→↔, ↕→↕, ⤢ → ⤡, ⤡ → ⤢, ↺ → ↺, ↻ → ↻ **E** → **E**, **B** → −**B**.

Concerning the refractive index n, we write it as the sum of two terms: a zeroth order term n_0 invariant with respect to P and T, and a term linear or bilinear with respect to the fields. Note that this second term is multiplied by a factor (–1) under a P or a T transformation, to take into account that both P and T symmetries are violated and that P^2 and T^2 amount to no transformation.

2.1 Transverse magnetic or electric field

Let us start our study by a pictorial analysis of light propagation along the z-axis in a centro-symmetric medium with interactions that violate P and T, where a magnetic field $\mathbf{B}_0$ is applied along the x-axis (see Fig. 1). In the case of a +45° polarized light (a), the refractive index can be written as $n_+ = n_0 + c_+ B_0^2$, while in the case of a −45° polarized light (d), it becomes $n_- = n_0 + c_- B_0^2$. Since the interactions violate T, experiment (a) becomes (b) under time-inversion. If we rotate (b) of an angle 180° about y-axis, we obtain (c) which is equivalent to (d) taking into account T-violation. We finally deduce that $c_+ = -c_-$ and $n_+ - n_- = 2c_+ B_0^2$. This is the optical signature of a Jones birefringence [15] proportional to B_0^2.

It is straightforward to show that our pictorial analysis gives the same result under space-inversion, and also that the same effect exists if one replaces the **B** field with an **E** field.

If one takes into consideration linear effects with respect to the transverse magnetic field **B** instead of quadratic ones, one obtains under PT transformation that $c_+ = -c_+$ (see Fig. 2), that obviously means that $c_+ = 0$. Starting from experiment (d) of Figure 1, one also obtains that $c_- = 0$. One obtains the same result when replacing the **B** field with an **E** field. Therefore, no Jones birefringence effect linear with respect to the field can exist.

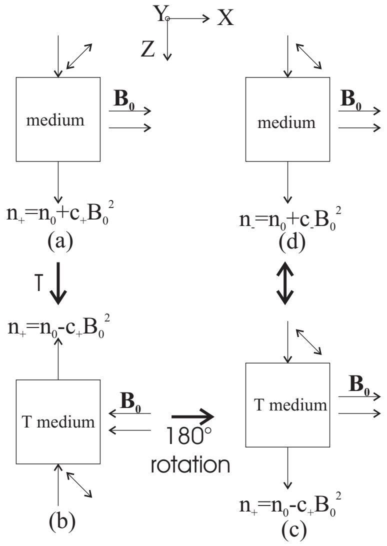

Fig. 1. Pictorial analysis of a magnetic quadratic effect under a T transformation.

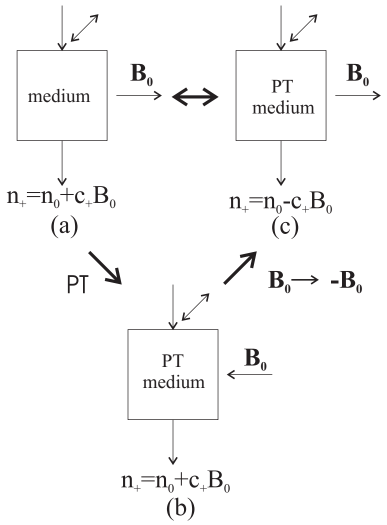

Fig. 2. Pictorial analysis of a magnetic linear effect under a PT transformation.

2.2 Longitudinal magnetic or electric field

If the field is longitudinal the optical eigenmodes [13] of light are the circular ones. In Figure 3 we show that a circular birefringence may exist in the case of a longitudinal electric field, since $c_+ = -c_-$. This is the optical effect treated in reference [12]. Using the same method, one can show that in the case of a longitudinal **B** field there is no effect since $c_+ = c_- = 0$. In Figure 4 we also show that no quadratic effect exists.

2.3 Transverse electric and magnetic fields

Let us now look for bilinear effects proportional to BE. If **E** and **B** are parallel (see Fig. 5), one obtains that

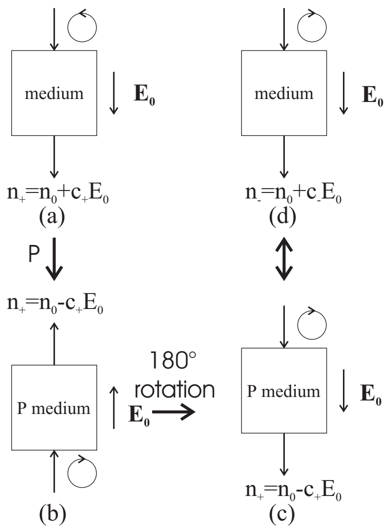

Fig. 3. Pictorial analysis of an electric linear effect under a P transformation for circularly polarized light.

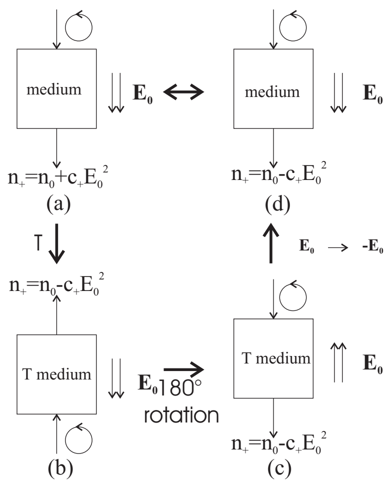

Fig. 4. Pictorial analysis of an electric quadratic effect under a T transformation for circularly polarized light.

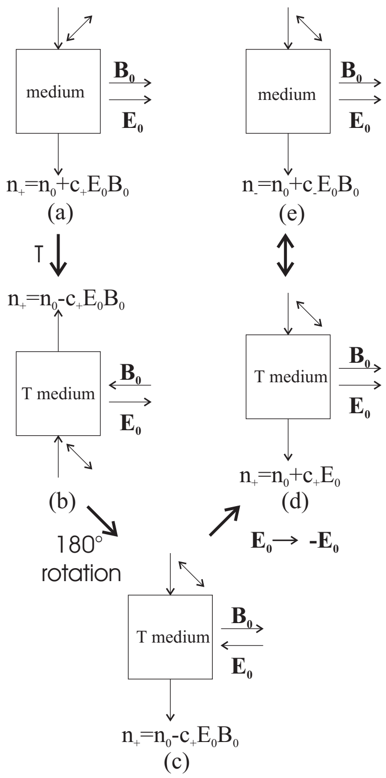

Fig. 5. Pictorial analysis of a bilinear magneto-electric effect under a T transformation, for parallel fields orthogonal to the light propagation direction.

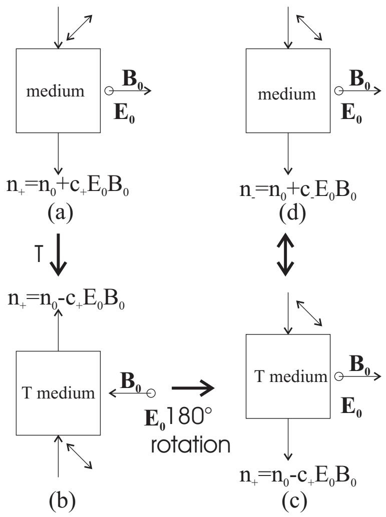

Fig. 6. Pictorial analysis of a bilinear magneto-electric effect under a T transformation, for perpendicular fields both orthogonal to the light propagation direction.

$c_+ = c_-$ both under P and T transformations. Thus the corresponding optical effect is a linear birefringence with one of the axis given by the two fields direction.

If **E** and **B** are perpendicular (see Fig. 6), one obtains that $c_+ = -c_-$ both under P and T transformation. Thus the corresponding optical effect is again a Jones birefringence.

2.4 Longitudinal electric and magnetic fields

If both electric and magnetic fields are parallel to the direction of light propagation, the optical eigenmodes [13] are once again the circular states. Figure 7 shows that a P transformation yields $c_+ = c_-$, hence no circular birefringence appears.

Applying a T transformation to the same system gives no further information ($c_+ = c_+$).

2.5 Longitudinal E (or B) and transverse B (or E)

To complete the study, we now address the case of a longitudinal electric field together with a transverse magnetic field. For circular polarizations, a P transformation gives $c_+ = c_-$ (see Fig. 8). One can show similarly that a T transformation adds no further information ($c_+ = c_+$). Therefore, no circular birefringence exists in this case.

One can also show that the same result holds if one exchanges the roles of **E** and **B**.

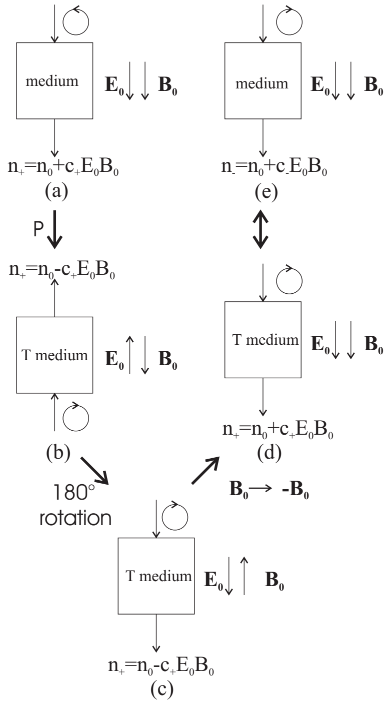

Fig. 7. Pictorial analysis of a bilinear magneto-electric effect under a P transformation, for fields both parallel to the light propagation direction.

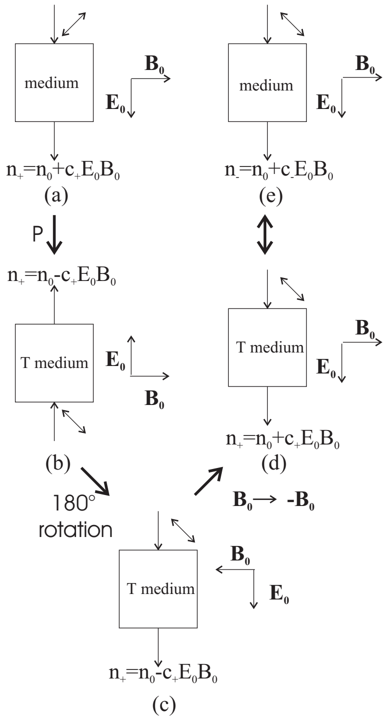

Fig. 9. Pictorial analysis of a bilinear magneto-electric effect under a P transformation, for a longitudinal electric field and a transverse magnetic field. Light is linearly polarized at 45° of the magnetic field.

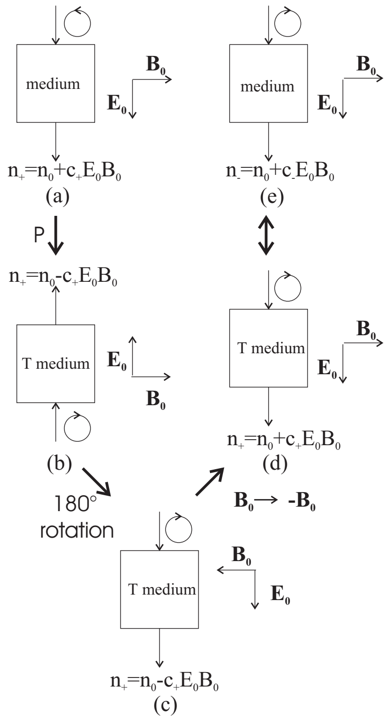

Fig. 8. Pictorial analysis of a bilinear magneto-electric effect under a P transformation, for a longitudinal electric field and a transverse magnetic field. Light is circularly polarized.

Table 1. Jones birefringence results.

JB	E_T or B_T	n
	$E_T\ E_T$ or $B_T\ B_T$ or $E_T \perp B_T$	y
	$E_T \parallel B_T$	n
	$E_L\ B_T$ or $E_T\ B_L$	n

Table 2. Circular birefringence results.

CB	E_L	y
	B_L	n
	$E_L\ E_L$ or $B_L\ B_L$	n
	$E_L\ B_L$	n
	$E_L\ B_T$ or $E_T\ B_L$	n

Finally, let us study the propagation of 45° linearly polarized light in the same field configuration. Figure 9 also yields $c_+ = c_-$, which implies that no Jones birefringence exists, but our analysis cannot exclude a linear birefringence.

It is worth stressing that these effects change sign when light is reflected back, since this corresponds to a change of sign for the light k vector without any change of the polarization direction i.e. a change from +45° to −45° polarization with respect to our pictorial analysis.

None of these new optical effects can exist in a standard CP and T non violating medium, and therefore they represent an unambiguous signature of such a violation.

Finally, we summarize our results in Tables 1 and 2, where the index T (resp. L) indicates that the field is transverse (resp. longitudinal).

3 Model calculation for quantum vacuum

The simplest medium where to perform a model calculation to search for the new optical effects found by the pictorial analysis is quantum vacuum.

In the presence of a magnetic and/or electric field, the vacuum becomes a non-linear medium, described by the

Heisenberg-Euler Lagrangian [16]:

$$L_{HE} = \frac{1}{2}F + bF^2 + 7bG^2 \quad (1)$$

where $F = \varepsilon_0 E^2 - B^2/\mu_0$ is C, P, T invariant and $G = \sqrt{\varepsilon_0/\mu_0}\,\mathbf{E}\cdot\mathbf{B}$ is not P, T invariant.

To perform a model calculation, let us introduce in the L_{HE} Lagrangian terms that violate P and T while being Lorentz invariant. Only terms proportional to G and FG fulfil these two conditions. Actually, since the standard model predicts a very small but non zero EDM for the electron and the positron, such terms in the Lagrangian describing the propagation of light in vacuum should in principle exist.

Since it is easier to produce a high magnetic energy density ($\mathcal{E}_{mag}/\mathcal{E}_{ele} = B^2c^2/E^2 = 9\times 10^4$ for $B = 1$ T and $E = 1$ MV/m), we will study the effect of a permanent transverse magnetic field $\mathbf{B}_0 = B_0\widehat{\mathbf{x}}$.

To study the propagation of light with the new Lagrangian

$$\begin{aligned} L = {} & \frac{1}{2}\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right) + a\sqrt{\frac{\varepsilon_0}{\mu_0}}\,(\mathbf{E}\cdot\mathbf{B}) \\ & + b\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right)^2 + 7b\frac{\varepsilon_0}{\mu_0}(\mathbf{E}\cdot\mathbf{B})^2 \\ & + d\sqrt{\frac{\varepsilon_0}{\mu_0}}\,(\mathbf{E}\cdot\mathbf{B})\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right) \end{aligned} \quad (2)$$

that is not P, T invariant, we will follow the method developed in reference [18]. We deduce from this Lagrangian:

$$\begin{aligned} \mathbf{D} \equiv \frac{\partial L}{\partial \mathbf{E}} = {} & \varepsilon_0\mathbf{E} + a\sqrt{\frac{\varepsilon_0}{\mu_0}}\mathbf{B} + 4b\varepsilon_0\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right)\mathbf{E} \\ & + \frac{14b\varepsilon_0}{\mu_0}(\mathbf{E}\cdot\mathbf{B})\,\mathbf{B} + d\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right)\mathbf{B} \\ & + 2\varepsilon_0 d\sqrt{\frac{\varepsilon_0}{\mu_0}}\,(\mathbf{E}\cdot\mathbf{B})\,\mathbf{E} \end{aligned} \quad (3)$$

$$\begin{aligned} \mathbf{H} \equiv -\frac{\partial L}{\partial \mathbf{B}} = {} & \frac{\mathbf{B}}{\mu_0} - a\sqrt{\frac{\varepsilon_0}{\mu_0}}\mathbf{E} + \frac{4b}{\mu_0}\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right)\mathbf{B} \\ & - \frac{14b\varepsilon_0}{\mu_0}(\mathbf{E}\cdot\mathbf{B})\,\mathbf{E} - d\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right)\mathbf{E} \\ & + \frac{2d}{\mu_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,(\mathbf{E}\cdot\mathbf{B})\,\mathbf{B}. \end{aligned} \quad (4)$$

Among all the terms in equations (3) and (4), there are terms at the frequencies 0, ω, and 2ω. We are just interested in the ω component, thus, removing terms at the frequencies 0 and 2ω, we obtain:

$$\begin{aligned} \mathbf{D}_\omega = {} & \varepsilon_0\left(1 - \frac{4b}{\mu_0}B_0^2\right)\mathbf{E}_\omega + \frac{14b\varepsilon_0}{\mu_0}(\mathbf{E}_\omega\cdot\mathbf{B}_0)\,\mathbf{B}_0 \\ & + \sqrt{\frac{\varepsilon_0}{\mu_0}}\left(a - \frac{d}{\mu_0}B_0^2\right)\mathbf{B}_\omega - \frac{2d}{\mu_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}(\mathbf{B}_0\cdot\mathbf{B}_\omega)\,\mathbf{B}_0 \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{H}_\omega = {} & \frac{1}{\mu_0}\left(1 - \frac{4b}{\mu_0}B_0^2\right)\mathbf{B}_\omega - \frac{8b}{\mu_0^2}(\mathbf{B}_0\cdot\mathbf{B}_\omega)\,\mathbf{B}_0 \\ & + \sqrt{\frac{\varepsilon_0}{\mu_0}}\left(-a + \frac{d}{\mu_0}B_0^2\right)\mathbf{E}_\omega + \frac{2d}{\mu_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}(\mathbf{E}_\omega\cdot\mathbf{B}_0)\,\mathbf{B}_0 \end{aligned} \quad (6)$$

Since $\mathbf{B}_0 = B_0\widehat{\mathbf{x}}$ and $\mathbf{k} = (n\omega/c)\widehat{\mathbf{k}} = (n\omega/c)\widehat{\mathbf{z}}$, we can write

$$\mathbf{D}_\omega = \varepsilon(B_0)\mathbf{E}_\omega + \psi_{DB}(B_0)\mathbf{B}_\omega \quad (7)$$

$$\mathbf{H}_\omega = \mu^{-1}(B_0)\mathbf{B}_\omega + \psi_{HE}(B_0)\mathbf{E}_\omega \quad (8)$$

with

$$\varepsilon = \varepsilon_0\begin{pmatrix} 1 + \frac{10b}{\mu_0}B_0^2 & 0 & 0 \\ 0 & 1 - \frac{4b}{\mu_0}B_0^2 & 0 \\ 0 & 0 & 1 - \frac{4b}{\mu_0}B_0^2 \end{pmatrix}, \quad (9)$$

$$\psi_{DB} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\begin{pmatrix} a - \frac{3d}{\mu_0}B_0^2 & 0 & 0 \\ 0 & a - \frac{d}{\mu_0}B_0^2 & 0 \\ 0 & 0 & a - \frac{d}{\mu_0}B_0^2 \end{pmatrix}, \quad (10)$$

$$\mu^{-1} = \frac{1}{\mu_0}\begin{pmatrix} 1 - \frac{12a}{\mu_0}B_0^2 & 0 & 0 \\ 0 & 1 - \frac{4a}{\mu_0}B_0^2 & 0 \\ 0 & 0 & 1 - \frac{4a}{\mu_0}B_0^2 \end{pmatrix}, \quad (11)$$

$$\psi_{HE} = -\sqrt{\frac{\varepsilon_0}{\mu_0}}\begin{pmatrix} a - \frac{3d}{\mu_0}B_0^2 & 0 & 0 \\ 0 & a - \frac{d}{\mu_0}B_0^2 & 0 \\ 0 & 0 & a - \frac{d}{\mu_0}B_0^2 \end{pmatrix}. \quad (12)$$

We will use Maxwell's equations in the classical limit:

$$\nabla\times\mathbf{E}_\omega = -\frac{\partial}{\partial t}\mathbf{B}_\omega, \qquad \nabla\cdot\mathbf{B}_\omega = 0,$$
$$\nabla\cdot\mathbf{D}_\omega = 0, \qquad \nabla\times\mathbf{H}_\omega = \frac{\partial}{\partial t}\mathbf{D}_\omega. \quad (13)$$

We assume the existence of plane wave eigenmodes with refractive index n:

$$\mathbf{E}_\omega(\mathbf{r},t) = \mathbf{E}_{\omega_0} e^{i\omega(\frac{n}{c}\widehat{\mathbf{k}}.\mathbf{r} - t)} \quad (14)$$

$$\begin{pmatrix} n^2\left(\frac{4b}{\mu_0}B_0^2-1\right)+2+\frac{10b}{\mu_0}B_0^2 & \frac{2dn}{\mu_0}B_0^2 & 0 \\ \frac{2dn}{\mu_0}B_0^2 & n^2\left(\frac{12b}{\mu_0}B_0^2-1\right)+2-\frac{4b}{\mu_0}B_0^2 & 0 \\ 0 & 0 & 2-\frac{4b}{\mu_0}B_0^2 \end{pmatrix}\mathbf{E}_\omega=\mathbf{E}_\omega \quad (17)$$

$$\begin{pmatrix} n^2\left(\frac{4b}{\mu_0}B_0^2-1\right)+2+\frac{10b}{\mu_0}B_0^2 & \frac{2dn}{\mu_0}B_0^2 \\ \frac{2dn}{\mu_0}B_0^2 & n^2\left(\frac{12b}{\mu_0}B_0^2-1\right)+2-\frac{4b}{\mu_0}B_0^2 \end{pmatrix}\mathbf{E}_\omega=\mathbf{E}_\omega \quad (18)$$

which, combined to the first Maxwell equation, yields $\mathbf{B}_\omega=(n/c)\widehat{\mathbf{k}}\times\mathbf{E}_\omega=(n/c)\Phi.\mathbf{E}_\omega$ with $\Phi.=\widehat{\mathbf{k}}\times.$ The last Maxwell's equation gives:

$$\left[\left(\frac{n}{c}\right)^2\Phi.\mu^{-1}\Phi.+\frac{n}{c}(\Phi.\psi_{HE}.+\psi_{DB}\Phi.)+\varepsilon.+\varepsilon_0\right]\mathbf{E}_\omega=\varepsilon_0\mathbf{E}_\omega. \quad (15)$$

Since $\widehat{\mathbf{k}}=\widehat{\mathbf{z}}$, we have

$$\Phi=\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (16)$$

and so

see equation (17) above.

Because we are only interested in the polarization plane, we restrict our analysis to the transverse directions:

see equation (18) above.

Let us first of all note that the a coefficient has cancelled out, which indicates that the lowest order P, T violating term does not contribute to the propagation of light.

If we set $d=0$, we recover the matrix of the Cotton-Mouton effect, while the non-diagonal terms can be interpreted in terms of a magnetic Jones birefringence.

The eigenvectors of this matrix are ($n=1+\mathcal{O}(B_0^2)$)

$$\mathbf{e}_\pm=\begin{pmatrix} 3b\pm\sqrt{(3b)^2+(2d)^2} \\ 2d \end{pmatrix}. \quad (19)$$

In order to identify the relative contributions of linear and Jones birefringence, we will now use the formalism of the Jones matrices [17], as in [18]. The Jones N matrices for linear and Jones birefringence are respectively $N_{CM}=g_0\begin{pmatrix} i & 0 \\ 0 & -i\end{pmatrix}$ and $N_J=g_J\begin{pmatrix} 0 & i \\ i & 0\end{pmatrix}$. Thus, the Jones matrix M writes

$$M\equiv\exp[(N_{CM}+N_J)z]=\begin{pmatrix} B+g_0A & Ag_J \\ Ag_J & B-g_0A \end{pmatrix} \quad (20)$$

with

$$A=i\frac{\sin\left(\sqrt{g_0^2+g_J^2}z\right)}{\sqrt{g_0^2+g_J^2}}$$

and

$$B=\cos\left(\sqrt{g_0^2+g_J^2}z\right).$$

Its eigenvectors are

$$\mathbf{g}_\pm=\begin{pmatrix} g_0\pm\sqrt{g_0^2+g_J^2} \\ g_J \end{pmatrix}. \quad (21)$$

Since $\mathbf{g}_\pm$ represent electric fields that propagate unchanged, they must be parallel to the eigenvectors $\mathbf{e}_\pm$ of equation (18). From this identification, we deduce

$$g_J=\frac{2d}{3b}g_0. \quad (22)$$

We can then determine g_0 by setting g_J to zero. The M matrix becomes $M=\begin{pmatrix} e^{ig_0z} & 0 \\ 0 & e^{-ig_0z}\end{pmatrix}$ with eigenmodes $\widehat{\mathbf{x}}$ and $\widehat{\mathbf{y}}$. The phase difference between the two modes is $2\arctan(g_0z)\simeq 2g_0z$. The QED calculation from the Heisenberg-Euler Lagrangian yields values for the index of refraction

$$n_x=1+\frac{7b}{2}\frac{B_0^2}{\mu_0},\quad n_y=1+2b\frac{B_0^2}{\mu_0}, \quad (23)$$

so that the phase difference between the two modes also writes $(3bB_0^2/2\mu_0)kz$. Finally one gets $g_0=(3bk/4\mu_0)B_0^2$ and $g_J=(kd/2\mu_0)B_0^2$. Since $n_+-n_-=2g_J/k$, we find

$$\Delta n=n_+-n_-=\frac{d}{\mu_0}B_0^2. \quad (24)$$

Thus the effect predicted by the pictorial analysis of Figure 1 exists, at least in principle, in quantum vacuum.

As for the effect of Figure 3, i.e. the circular birefringence proportional to the E field, it cannot exist in a quantum vacuum since no term trilinear with respect to the fields can exist in the quantum vacuum Lagrangian because of the Lorentz invariance.

To recover the complete set of effects, one does not need to perform the same lengthy calculation for all the cases. As shown in reference [19], one can use the fact that vacuum is Lorentz invariant, perform an appropriate Lorentz transformation to calculate the optical effects where only one field exists and then transform back to the original system.

In the case of two crossed fields $\mathbf{E}_0$ and $\mathbf{B}_0$ in a frame K, using Lorentz transformations [20] one can show that in a frame K' moving with velocity $\beta = \mathbf{v}/c$ relative to K only a magnetic field $\mathbf{B'}_0$ exists if

$$\mathbf{v} = c\frac{\mathbf{E}\times\mathbf{B}}{B^2} \tag{25}$$

and therefore $\beta = E/B$. The value of $\mathbf{B}'$ is

$$\mathbf{B}' = \sqrt{1-\beta^2}\mathbf{B}. \tag{26}$$

In reference [19] it is shown that, once n and n' can be written as $n = 1 + \delta n$ with $\delta n \ll 1$ and $n' = 1 + \delta n'$ with $\delta n' \ll 1$, if the $\mathbf{k}$ wave vector of light and $\mathbf{v}$ are parallel, δn can be written, up to first order with respect to $\delta n'$, as

$$\delta n = \delta n'\frac{1-\beta}{1+\beta}. \tag{27}$$

If $\mathbf{k}$ and $\mathbf{v}$ are antiparallel, one has simply to change the sign of β.

Let us finally calculate the value of n using the expressions just introduced. In the frame K', we are interested in the Jones birefringence due to the P, T violation. We can therefore write that

$$\delta n'_+ = \eta_+ B_0'^2 = \eta_+\left(1-\beta^2\right)B_0^2 \tag{28}$$

and

$$\delta n_+ = \eta_+(1-\beta)^2 B_0^2 = \eta_+\left(B_0^2 - 2E_0B_0 + E_0^2\right). \tag{29}$$

For the same reason, we can write

$$\delta n_- = \eta_-(1-\beta)^2 B_0^2 = \eta_-(B_0^2 - 2E_0B_0 + E_0^2). \tag{30}$$

Thus, if $\eta_+ = -\eta_-$ i.e. if a Jones effect proportional to B_0^2 exists, we have demonstrated that a Jones effect proportional to E_0^2 and the magneto-electric Jones birefringence proportional to E_0B_0 must also exist.

4 Conclusion

Pictorial analysis introduced by de Figueiredo and Raab [14] is a very powerful tool to study optical effects also in non standard media. In the case of P, T violating interactions and centro-symmetric media, we have shown that new optical effects can be predicted, such as a Jones birefringence proportional to B_0^2. With respect to C, P and T transformations, a P, T violating interaction and an electric field $\mathbf{E}$ generally correspond to a physical object that transforms as a magnetic field $\mathbf{B}$. Similarly, a P, T violating interaction and a magnetic field $\mathbf{B}$ correspond to a physical object that transforms as an electric field $\mathbf{E}$. This suggests that the roles of $\mathbf{E}$ and $\mathbf{B}$ are inverted in the presence of a P, T violating interaction. One therefore expects that a Pockels effect induced by a $\mathbf{B}$ field exists in non-centro-symmetric media in the presence of a P, T violation. On the other hand a true chiral medium and a magnetic field are a physical object that violates P and T. We can therefore predict that in true chiral media a circular birefringence should exist in the presence of both a longitudinal $\mathbf{B}$ and a longitudinal $\mathbf{E}$ field.

The calculation of a quantitative value for all these new phenomena is out of the scope of this paper. One would need the values of P, T violating hyperpolarizabilities to perform such a calculation, and as far as we know these values do not exist in literature. However, values of P, T violating polarizability η' can be found in literature. For xenon in reference [21] one finds $\eta' = -4.5\times10^{-2}\,d_e$ and $\eta' = -1.07\,d_e$ for radon, where d_e is the electron EDM. Larger values of η' are reported in [1] for cesium, $\eta' \simeq 100\,d_e$, and francium, $\eta' \simeq 1000\,d_e$. In reference [12] one can find an estimation of the rotation angle of the polarization plane induced by a longitudinal electric field in the case of a light beam resonant with a particular transition of cesium gas.

Anyway, a rough estimation of the value of expected effects can be obtained by taking into consideration the fact that quantum mechanical expressions of standard hyperpolarizabilities involve products of matrix elements of the electric dipole moment or products of matrix elements of the electric dipole moment and of the magnetic dipole moment [15]. In P, T violating hyperpolarizabilities the matrix elements of at least one of these (electric or magnetic) dipole moments is replaced by the matrix elements of the P, T violating EDM. The value of corresponding hyperpolarizabilities with respect to the standard one therefore scales as the ratio between the standard elements and those involving the P, T violating EDM. Standard electric dipole elements are on the order of ea_0, where e is the electron charge and a_0 the Bohr radius, corresponding to 5×10^{-9} e.cm, whereas non standard ones are on the order of d_e which current value is less than 1.6×10^{-27} e.cm. The effects are therefore expected to be very weak and experiments very challenging. Polarization rotation induced by a longitudinal electric field is linear with respect to the field amplitude and, in principle, it looks more interesting for an electro-optical search of P, T violating interaction than the new magneto-optical effect that we present here that is quadratic with respect to the field amplitude. In practice, electric fields are much more difficult to produce than magnetic fields. To calculate the expected optical effect, in reference [12] authors assume fields of the order of 10^6 V/m. As for energy density, this high field corresponds of only about 3×10^{-3} T . Magnetic field effect therefore is worth further studies since magnets providing a transverse magnetic field higher than 1 T are of common use in Cotton-Mouton measurements (see e.g. [22] for permanent magnets, and [23] for superconductive magnets).

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References

1. J.S.M. Ginges, V.V. Flambaum, Phys. Rep. **397**, 63 (2004)
2. E.M. Purcell, N.F. Ramsey, Phys. Rev. **78**, 807 (1950)
3. B.C. Regan, E.D. Commins, C.J. Schmidt, D. DeMille, Phys. Rev. Lett. **88**, 071805 (2002)
4. P.G. Harris, C.A. Baker, K. Green, P. Iaydjiev, D.J.R. May, J.M. Pendlebury, D. Shiers, K.F. Smith, M. van der Grinten, P. Geltenbort, Phys. Rev. Lett. **82**, 904 (1999)
5. M.V. Romalis, W.C. Griffith, J.P. Jacobs, E.N. Fortson, Phys. Rev. Lett. **86**, 2505 (2001)
6. R.D. Peccei, H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977)
7. L.D. Barron, A.D. Buckingham, Acc. Chem. Res. **34**, 781 (2001)
8. L.D. Barron, Nature **405**, 895 (2000)
9. S.K. Lamoreaux, Phys. Rev. A **66**, 022109 (2002)
10. N.B. Baranova, Y.V. Bogdanov, B.Y. Zel'dovich, Usp. Fiz. Nauk **123**, 349 (1977)
11. O.P. Sushkov, F.F. Flaumbaum, Zh. Eksp. Teor. Fiz. **75**, 1208 (1978)
12. V.G. Baryshevsky, D.N. Matsukevich, Phys. Rev. A **66**, 062110 (2002)
13. D. Budker, W. Gawlik, D.F. Kimball, S.M. Rochester, V.V. Yashchuk, A. Weis, Rev. Mod. Phys. **74**, 1153 (2002)
14. I.M.B. de Figueiredo, R.E. Raab, Proc. R. Soc. Lond. A **369**, 501 (1980)
15. E.B. Graham, R.E. Raab, Proc. R. Soc. Lond. A **390**, 73 (1983)
16. H. Euler, B. Kockel, Naturwissenschaften **23**, 246 (1935); W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)
17. R.C. Jones, J. Opt. Soc. Am. **38**, 671 (1948)
18. G.L.J.A. Rikken, C. Rizzo, Phys. Rev. A **63**, 012107 (2001)
19. C. Rizzo, G.L.J.A. Rikken, Phys. Scripta **71**, C5 (2005)
20. R. Becker, *Electromagnetic fields and interactions* (Dover, New York, 1982)
21. B. Ravaine, M.G. Kozlov, A. Derevianko, Phys. Rev. A **72**, 012101 (2005)
22. F. Brandi, F. Della Valle, P. Micossi, A.M. De Riva, G. Zavattini, F. Perrone, C. Rizzo, G. Ruoso, J. Opt. Soc. B **15**, 1278 (1998)
23. M. Bregant, G. Cantatore, S. Carusotto, R. Cimino, F. Della Valle, G. Di Domenico, U. Gastaldi, M. Karuza, E. Milotti, E. Polacco, G. Ruoso, E. Zavattini, G. Zavattini, Chem. Phys. Lett. **410**, 288 (2005)